Neutrinoless double beta decay in the dualized standard model

José Bordes*

Departament Fisica Teorica, Universitat de Valencia, calle Dr. Moliner 50, E-46100 Burjassot (Valencia), Spain

Chan Hong-Mo[†]

Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom

Ricardo Gallego[‡]

Departament Fisica Teorica, Universitat de Valencia, calle Dr. Moliner 50, E-46100 Burjassot (Valencia), Spain

Tsou Sheung Tsun[§]

Mathematical Institute, University of Oxford, 24-29 St. Giles', Oxford, OX1 3LB, United Kingdom

The dualized standard model offers a *raison d'être* for 3 fermion generations and an explanation for their distinctive mass and mixing patterns, reproducing to a reasonable accuracy the empirical mixing matrix and mass spectrum for both quarks and leptons in terms of just a few parameters. With its parameters thus fixed, the result is a highly predictive framework. In particular, it is shown that it gives explicit parameter-free predictions for neutrinoless double beta decays. For ^{76}Ge, it predicts a half-life of 10^{31}–10^{32} yr, which satisfies the present experimental lower bound of 1.8×10^{25} yr.

Neutrinoless double beta decays of the type

$$A \rightarrow B + 2e^-, \qquad (1)$$

in which a nucleus $A = (Z,N)$ converts into a nucleus $B = (Z+2, N-2)$ emitting $2e^-$ with no accompanying neutrinos, has long been recognized as a most promising probe for possible lepton-number violation [1]. It is also the most likely test to decide whether the neutrino is a Majorana particle, since the best known possibility for the decay is the exchange of such neutrinos. For this reason, high sensitivity experiments have already been done giving very stringent bounds on the decay rate, of which the tightest so far is from the Heidelberg-Moscow experiment searching for the decay

$$^{76}\text{Ge} \rightarrow {}^{76}\text{Se} + 2e^-, \qquad (2)$$

which after a 31 kg yr run gives a limit on the half-life [2]:

$$\tau(^{76}\text{Ge}) > 1.8 \times 10^{25} \text{ yr}(90\% \text{ C.L.}). \qquad (3)$$

Further effort with this experiment, it is claimed, can reach limits of up to 10^{27} yr in one year, and up to 10^{29} yr in ten years of running, so that an improvement of several orders of magnitude is foreseeable in the near future.

The decay (1) violates lepton-number by 2 units and is thus forbidden in the conventional version of the standard model. However, lepton number is conserved there only by a global symmetry which would be broken at some level in almost any extension to the model and would lead to neutrinoless double beta decays of the type (1). It is thus incumbent upon advocates of any extension to the standard model to check whether their proposal predicts decay rates for Eq. (1) which are first, consistent with present bounds, and second, accessible to future experiment.

In particular, the dualized standard model (DSM) [3] which we ourselves advocate has to be subjected to such a test. This DSM scheme purports to extend the conventional version of the standard model in such a way as (i) to offer an explanation for the existence of 3 and only 3 fermion generations, (ii) to deduce the qualitative features of fermion mixing and the hierarchical fermion mass spectrum, and (iii) to allow a systematic calculation of the mixing parameters both for quarks and leptons giving results in general agreement with experiment [3,4]. However, in the DSM explanation for neutrino oscillations, the neutrinos acquire their very small masses through the seesaw mechanism [5] with the introduction of right-handed singlets. Lepton-number violation is thus implied giving decays of type (1) so that it is incumbent upon us to check its predictions in this against experiment. In fact, as we shall show, the model is so constrained by its calculation of the fermion mass and mixing parameters [4] that its prediction for Eq. (1) is now entirely parameter-free and explicit. For Eq. (3), in particular, it gives

$$\tau(^{76}\text{Ge}) = 10^{31} - 10^{32} \text{ yr}, \qquad (4)$$

with the uncertainty coming mostly from the present empirical uncertainty in the mass m_3 of the heaviest neutrino. Comparing this result with Eq. (3), we conclude first, that the DSM scheme survives the present empirical bound, which is in itself nontrivial since the prediction is parameter-free, and second, that the prediction is between one and two orders of magnitude below the sensitivity range of present planned experiments.

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*Also at IFIC, Centro Mixto Universitat de Valéncia-CSIC. Email address: jose.m.bordes@uv.es

[†]Email address: chanhm@v2.rl.ac.uk

[‡]Email address: gallegor@titan.ific.uv.es

[§]Email address: tsou@maths.ox.ac.uk

In what follows, we shall detail how the above result (4) is derived, finishing with a discussion of its possible implications.

First, a few words about the general features of the DSM scheme which lead to fermion mixing and fermion mass hierarchy. Using theoretical results derived earlier [6,7] a candidate for the ''horizontal symmetry'' of generations is identified as the dual to color $SU(3)$ [3] which is naturally broken [8] giving 3 and only 3 generations as a result. Duality suggests also the mechanism for breaking the symmetry [3] leading to fermion mass matrices with only one nonzero eigenvalue (rank-one). At the tree level, this means zero masses for the 2 lower generations and no mixing between up and down states. With loop corrections, however, the mass matrix changes its orientation in generation space (rotates) with changing energy scales. As a result, mass ''leaks'' from the heaviest into the 2 lower generations, giving the characteristic hierarchical mass spectrum observed. Further, mixing occurs between up and down flavor states, with the mixing matrix elements given as direction cosines between the two triads of mass eigenvectors at the two scales corresponding to respectively the up and down states. The framework depends on several parameters related to the vacuum expectation values (VEV's) and Yukawa couplings of the (dual color) symmetry-breaking Higgs bosons, loops of which are what drive the mass matrix rotation. Of these parameters those 3 relevant to the mass and mixing patterns were fitted to m_c/m_t, m_μ/m_τ, and the Cabibbo angle. Given then the masses of the heaviest generation, one predicts the masses of the other quarks and charged leptons together with the remaining quark mixing angles all of which are in general agreement with experiment. For more details the reader is referred to e.g. [4].

When applying the above mechanism to neutrinos, however, giving just the physical mass m_3 of the heaviest neutrino is not enough since by the see-saw mechanism [5] the physical masses m_i of neutrinos depend on both their Dirac masses M_i and the right-handed neutrino mass B, thus: $m_i = M_i^2/B$, and it is on the Dirac mass matrix that mixing depends [9]. One inputs therefore also the physical mass m_2 of the second generation neutrino, but here, one finds that not all inputs for m_2 will work, for the DSM mechanism, as explained above, will give only a hierarchical mass spectrum. In practical terms, it means that of the current 4 admissible solutions for solar neutrinos [10], only the so-called vacuum and low solutions are admitted by DSM.[1] Inputting then $m_3^2 \sim 3 \times 10^{-3}$ eV^2 as indicated by atmospheric neutrino data and $m_2^2 \sim 10^{-10}$ eV^2 for the vacuum solar neutrino solution [10], all other required parameters being already

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[1]We note that although it was thought at one stage [11] that recent data from Superkamiokande exluded both the small mixing angles (SMA) and vacuum-low solutions at 95 percent confidence level, later more thorough anaylses of the global date [12] show that at present there is no reason to exlude any of the solutions [low mixing angle (LMA), SMA, vacuum-low] at any reasonable confidence level.
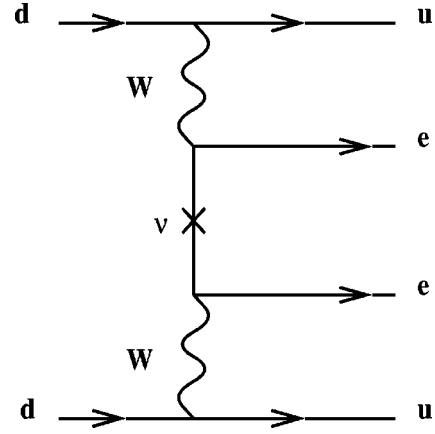

FIG. 1. Feynmann diagram illustrating the neutrinoless double beta decay.

fixed as explained above, one predicts with DSM [4] the lepton mixing matrix together with all the other neutrino masses m_1, M_i, $i = 1,2,3$, and B. In particular, one finds the mixing matrix elements:

$$U_{e3} \sim 0.07, \quad U_{\mu3} \sim 0.66, \tag{5}$$

a right-handed neutrino mass B of order of a few hundred TeV, and the mass m_1 of the lightest neutrino very small (as low as 10^{-15} eV!). These are all that is needed for our discussion here. In passing, we note that the predicted mixing angles (5) are both in excellent agreement with present experiment [10,13]. We shall return later to comment on the accuracy and reliability of the above predictions.

Next, turning to neutrinoless double beta decay, an explicit formula for the half-life of the process (1) has been worked out in full generality in, for example [14], for an effective Lagrangian with left and right handed fermionic currents, which formalism we shall mainly follow, since the theory involved here is basically contained in the standard model except for the addition of a right-handed neutrino component to allow for lepton number violation. In Fig. 1 we give the Feynman diagram for the process. As one can see, the emission of 2 W gauge bosons transforms the 2 d quarks into u quarks, and in turn the 2 neutrons into the corresponding protons. In each weak vertex an element of the Maki-Nakagawa-Sakata (MNS) mixing matrix [15] has to be included, and in the neutrino propagator it is the physical mass of the neutrino that enters. As a result, given that the physical neutrino masses are much smaller than the energies typically involved, the amplitude is proportional to

$$\langle m_\nu \rangle = \sum_j m_j U_{ej}^2, \tag{6}$$

which may be considered as an effective mass for the neutrino exchanged.

With these ingredients we have for the inverse half-life of a nucleus in a transition between 0^+ states:

$$\Gamma_{0\nu}(0^+ \to 0^+) = \frac{G_F^4 m_e^5}{16\pi^5} G_{01}(T) g_A^4 |\langle M_{Nucleus}\rangle|^2 \langle m_\nu \rangle^2.$$

(7)

(We have not shown explicitly the Fermi factor for the emitted electrons since it can be consistently substituted by a factor of two.)

The different factors appearing in Eq. (7) arise as follows. First, the Fermi constant raised to the 4th power comes from the two weak vertices. Second, there is the phase space factor $G_{01}(T)$ given by

$$G_{01}(T) = \frac{1}{15} T(T^4 + 10T^3 + 40T^2 + 60T + 30)$$

(8)

where T is the maximum kinetic energy attainable by the emitted electrons in units of the electron mass. Since for the cases of interest T is in the range (2,3), no further simplification in Eq. (8) is allowed, but T itself may be approximated by

$$T = \frac{M_A - (M_B + 2m_e)}{m_e} \sim 2\frac{m_n - m_p - m_e}{m_e}.$$

(9)

Third, the expectation values for the Hamiltonian in nuclear states ($\langle M_{Nucleus}\rangle$) involving the nuclear structure have been treated in the usual effective way, using a non-relativistic approximation for the nuclear motion and the impulse approximation for the interaction between leptons and nucleons. The nuclear interactions in the limit when the momentum transfer between nucleons is low compared to the nucleon mass is taken into account by three form factors describing the so-called Fermi, Gamow-Teller and tensor interactions. The values of these matrix elements have been calculated for various nuclei in different nuclear models [16]. The additional factor g_A is the effective form factor describing the interactions between the electroweak gauge bosons and the quarks inside the nucleons at zero momentum transfer which is appropriate for this case. Finally there is the square of the neutrino effective mass $\langle m_\nu \rangle$ already mentioned.

One notes that of all the factors appearing in Eq. (7), only the last $\langle m_\nu \rangle^2$ depends specifically on the DSM scheme through the neutrino masses and mixing angles. Given now that in the DSM, as explained before, the masses of neutrinos are hierachical, namely that $m_3 \gg m_2 \gg m_1$ dropping by at least 3 orders of magnitude from generation to generation, the sum in Eq. (6) is dominated entirely by the m_3 term despite the smallness of the MNS mixing element U_{e3}, giving

$$\langle m_\nu \rangle \sim m_3 (U_{e3})^2.$$

(10)

In other words, to a good approximation, the half-life for neutrinoless double beta decay depends in the end only on the mass m_3 of the heaviest neutrino mass eigenstate ν_3 and on the mixing element U_{e3} from the electron neutrino into this state. The former quantity m_3 is known to a fair accuracy (assuming the masses to be hierarchical) from, e.g., the Su-

perkamiokande experiment [10], while the latter quantity U_{e3} was calculated from the DSM scheme giving the value in Eq. (5). The half-life is therefore explicitly calculable.

In particular, let us apply the formula to the most promising example (2) for which a running experiment not only gave already a very non-trivial bound but can in the foreseeable future improve considerably on the present limits [2]. We note that in the nuclear matrix elements between states of the same spin-parity (0^+) the dominant contribution comes from the Gamow-Teller transition. This piece has been evaluated for instance in [16] using the pn renormalized quassiparticle random phase approximation (RQRPA) model which gives good results in heavy nuclei, resulting in the value for a nucleus of ^{76}Ge $\langle M_{Ge}\rangle = 0.28$ Gev. Then, with the axial-vector form factor conventionally taken as $g_A = 1.24$, we obtain, from Eq. (7),

$$\Gamma_{0\nu}(^{76}\text{Ge}) = 4.8 \times 10^{-39} \langle m_\nu \rangle^2 \text{ GeV}^{-1}.$$

(11)

Substituting then the DSM value of U_{e3} from Eq. (5) [4] and the presently allowed range $m_3^2 = 10^{-2} - 10^{-3}$ eV^2 for the (physical) mass of the heaviest neutrino [10,17], we obtain

$$\Gamma_{0\nu}(^{76}\text{Ge}) = (1-10) \times 10^{-64} \text{ GeV}.$$

(12)

Apart from the folding in of some minor numerical uncertainties in the DSM calculation which we shall now clarify, this corresponds to the range of half-life for ^{76}Ge quoted before in Eq. (3).

In the DSM calculation of the mixing parameters (5), besides the possible uncertainties in the scheme itself which we would not know how to estimate, the main imprecisions came from the empirical quantities used to determine the unknown parameters, namely, the quark masses and the Cabibbo angle. This gave a range of values for $U_{e3} = 0.063-0.073$ [4], corresponding to a spread in the predicted value of $\Gamma_{0\nu}$ only of about a factor 2. This means that if it were not for the uncertainties in the nuclear physics and in the present empirical value of m_3, the prediction of DSM on $\Gamma_{0\nu}$ could be made much more precise. The mixing element U_{e3} depends in principle also on the ratio m_2/m_3, which within the DSM framework is equivalent to a dependence on the Dirac mass M_3 of ν_3, but this dependence is weak, as demonstrated numerically in [9], so long as M_3 remains in the MeV range as in the quoted calculation [4]. More concisely, it can be shown with a little calculation that to a good approximation:

$$U_{e3} = 0.15 \sin\left(\theta - \sqrt{\frac{M_2}{M_3}}\right) - \frac{1}{2} \sqrt{\frac{M_3}{m_0}},$$

(13)

(where $\theta = \tan^{-1}\sqrt{2}$ and $m_0 = 1.25$ GeV, which are parameters of the model related to the boundary conditions of the renormalization group equation, namely the vacuum expectation values (VEV's) of dual Higgs bosons). Although the precise choice of M_3 hardly affects the prediction for the rate of neutrinoless double beta decays, we would nevertheless advocate our particular choice in [9,4] of $M_3 \sim 4$ MeV because this corresponds to a right-handed neutrino mass B of order 500 TeV, which happens also to be the symmetry-

breaking scale for the generation (dual color) symmetry in DSM as estimated from flavor changing neutral current (FCNC) effects [18] and the so-called post Grejsen-Zatsepin-Kuz'min (GZK) cosmic ray air showers [19]. One is reminded of a parallel situation in grand unified theories where it has been widely assumed that the mass B is of the same order of magnitude as the unification breaking scale, although the actual scale of order 10^{15} GeV there is very different from the 100 TeV scale advocated here for the DSM.

From Eq. (13), one sees that the mixing element U_{e3} in the DSM scheme is generically non-zero. This can be seen also from the fact that in DSM a corner element such as U_{e3} in a mixing matrix [whether Cabibbo-Kobayashi-Maskawa (CKM) or MNS] can be interpreted as the effect of torsion of a certain curve on the unit sphere which, though vanishing to first order in separation between up and down states and therefore small, has no reason to be zero for finite separation [20,4]. That U_{e3} is nonzero is what makes the DSM prediction for the rate of neutrinoless double beta decay non-vanishing, and thus in principle observable though not perhaps in the foreseeable future, and distinguishes it from models with exact bimaximal mixing which predicts the element U_{e3} to be precisely zero, in which case the experimental observation of the effect would be ruled out.

To advocates of the DSM scheme like ourselves the fact that its prediction for neutrinoless double beta decay survives existing bound is of course a relief, given that the calculation has no freedom and depends on no adjustable parameters. However, so long as the effect remains undiscovered, it is not a positive test for the validity of the scheme like the original tests on the fermion mass and mixing parameters [4]. Still it adds to the list of DSM predictions which have survived experimental bounds after the parameters of the scheme had been fixed. This now includes FCNC effects in meson mass differences and decays [18], post-GZK cosmic ray airshowers [19], $\mu - e$ conversions in nuclei and muonium [21], and lepton-flavor violation due to ''transmutation'' [22] in γe [23] and $e^{+}e^{-}$ [24] collisions, and in vector boson decays [25]. Together now with neutrinoless double beta decay, the total weight even of just these survival tests is beginning to look somewhat nontrivial. Moreover, since some of the effects predicted are well within present experimental sensitivity, there is good hope that they may be positively tested in the not too distant future.

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